Performance Estimation for Turbofans with and without Mixers

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A relatively simple equation set is developed to estimate the optimal performance of mixed-stream turbofans with component losses included. The set provides the specific fuel consumption and required bypass flow and fan pressure ratios to give a prescribed specific thrust for given flight conditions, component design limits, and engine core stream Mach number entering the mixer. Example results confirm the expectation (based on ideal cycle analysis) that the optimal specific fuel consumption and bypass ratio are nearly independent of the fan pressure ratio, provided that an optimal mixer is employed. Further, an engine designed for use in a mixedstream configuration only would best use a lower fan pressure ratio than would an engine deliberately designed for an unmixed exhaust.

Nomenclature

- a_0 = speed of sound
- e_c = compressor polytropic efficiency
- $e_{c'}$ = fan polytropic efficiency
- e_t = turbine polytropic efficiency
- \boldsymbol{F} = thrust
- = fuel-to-air ratio
- \boldsymbol{G} $=F/(\dot{m}_c+\dot{m}_F)a_0M_0$
- = fuel heating value
- $K = \text{mixer loss parameter, } \pi_m = 1 KM_7^2$
- M = Mach number
- \dot{m} = mass flow rate
- p = pressure
- = stagnation pressure p_t
- S = specific fuel consumption
- $S_{3'} = (p_{t3'}/p_{t5})^{(\gamma_g 1)/\gamma_g}$ T = temperature
- T_t = stagnation temperature
- = bypass ratio = \dot{m}_F/\dot{m}_c
- = ratio of specific heats upstream of burner and in fan
- = ratio of specific heats in turbine
- γ_9 = ratio of specific heats in mixer and nozzle
- $\eta_b = \text{burner efficiency}$
- $\eta_m = \text{shaft mechanical efficiency}$
- $\pi_b = p_{t4}/p_{t3}$
- $\begin{array}{lll} \pi_b &= p_{t4}/p_{t3} \\ \pi_c &= p_{t3}/p_{t2}, & \tau_c = T_{t3}/T_{t2} \\ \pi_{c'} &= p_{t3'}/p_{t2}, & \tau_{c'} = T_{t3'}/T_{t2} \\ \pi_d &= p_{t2}/p_{t0} \\ \pi_m &= p_{t8}/p_{t7} \\ \pi_n &= p_{t9}/p_{t8} \\ \pi_t &= p_{t5}/p_{t4}, & \tau_t = T_{t5}/T_{t4} \\ \tau &= T_{t3'}/T_{t5} \end{array}$

- $\tau_r = 1 + \frac{\gamma_c 1}{2} M_0^2$
- $au_{\lambda} = \frac{C_{pt}T_{t4}}{C_{pc}T_0} \frac{\text{stagnation enthalpy at entry to turbine}}{\text{ambient enthalpy}}$
- $B_1 = \left(\frac{\tau_{\lambda}}{\tau_r \tau_c} I\right) \frac{\tau_r \tau_c I}{\tau_r I}$
- $B_2 = (l+f)\tau_{\lambda}\tau_t + \alpha\tau_r\tau_{c'}$

$$B_3 = \left(I + \frac{\gamma_9 - I}{2} M_5^2\right) / \left(\frac{\gamma_9 - I}{2} \frac{M_5^2}{\Gamma_I} - \frac{\tau_{\lambda}}{\tau_{I}} + \tau_c C_I\right)$$

- $C_{I} = \pi_{b}^{(\gamma_{g}-1)/\gamma_{g}} \tau_{c}^{(\Gamma_{2}e_{c}-1)} \tau_{t}^{(\Gamma_{3}/e_{t}-1)} \tau_{c}^{(I-e_{c}'\Gamma_{2})}$
- $C_2 = (\pi_d \pi_b \pi_m \pi_n)^{(\gamma_g 1)/\gamma_g} \tau_r^{(\Gamma_2 1)} \tau_c^{(e_c \Gamma_2 1)} \tau_t^{(\Gamma_3/e_t 1)}$

$$C_3 = \left(l + \frac{f}{l + \alpha}\right) B_2 \left[l - \left(\frac{p_0}{p_{t9}}\right)^{(\gamma_9 - 1)/\gamma_9}\right] / (\tau_r - l) \left(l + \alpha + B_l\right)$$

- $\Gamma_I = \frac{\gamma_c I}{\gamma_c} \frac{\gamma_t}{\gamma_t I}$
- $\Gamma_2 = \frac{\gamma_c}{\gamma_c 1} \frac{\gamma_g 1}{\gamma_g}$
- $\Gamma_3 = \Gamma_1 \Gamma_2$

Subscripts

0-9 = station numbers (see Fig. 1)

= core stream

 \vec{F} = fan stream

Superscripts

()' = conditions in fan stream

Introduction

Mixed-stream turbofan engines, in which the bypass air and core stream air are mixed prior to expulsion from a common propelling nozzle (Fig. 1), are viewed as top candidates for the next generation of very-high-performance aircraft. Several benefits accrue from utilizing a common propelling nozzle, including the provision of aircraft cooling by the fan stream and the provision of afterburner flame stabilization by the hot core gas. In some cases, such mixing leads to thrust improvements from increases in overall propulsive efficiency. Mixing of the streams does lead to some penalties, however, primarily in the off-design flow-matching restriction brought about by the requirement of (near) static pressure matching at the mixer inlet splitter plate.

Preliminary design can illuminate many of the performance tradeoffs resulting from design configuration changes. Unfortunately even with a preliminary design, in which all of the loss mechanisms are included, the student engineer (if not the practicing engineer) can certainly find himself confronted with a bewildering array of input parameters from which it is difficult to determine meaningful trends. At the preliminary stage, conceptualization can be aided by not only invoking appropriate approximations and simplifications, but also by utilizing analytical methods to generate "optimal" solutions. Such optimal solutions have the advantage of not only leading

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the designer to at least the vicinity of a desirable configuration, but by simultaneously generating pairs of families of the related design (input) parameters, of reducing the required graphical representations of design parameters vs resultant performance.

In this paper, several definitions of "optimal" performance are used. Thus, when the overall engine performance is considered, the optimal performance is taken to be that corresponding to a minimum specific fuel consumption for a prescribed specific thrust. The specific thrust is itself defined as that existing at the prescribed design Mach number at which the related specific fuel consumption is to be evaluated. On the other hand, an optimal mixer is defined as that giving the maximum possible outlet stagnatin pressure for a prescribed inlet mass flow ratio and stagnation conditions.

A relatively simple example of an optimal method is provided in Ref. 1 (p. 241), where it is shown that a simple numerical search technique (programmable on a desk calculator) can be used to obtain the minimum specific fuel consumption for a given specific thrust and compressor pressure ratio (flight conditions and measures of real component performances, such as compresor efficiency and burner total pressure loss, must also be supplied). The case considered there is that of a separate (unmixed) stream turbofan and the results provide, in addition to the specific fuel consumption, the related bypass ratio and bypass pressure ratio. It is also found that, as would be intuitively expected, the required bypass ratio decreases and required bypass pressure ratio increases with the increase in the prescribed specific thrust.

In this paper, similar methodology is invoked in order to estimate the design performance of mixed-stream turbofans. The analysis is aided by the observation Ref. 1, (Prob. 5.20) that, for an ideal mixed stream turbofan incorporating an ideal constant-pressure mixer, the specific thrust is independent of the fan pressure ratio provided only that $p_{t3'}/p_{t5} > 1$. As a result of the independency of the specific thrust and the fan pressure ratio, a single equation can be obtained giving the specific thrust as a function of the bypass ratio. For the ideal case, the resulting equation is easily inverted to give an explicit expression for the bypass ratio required to deliver the specified specific thrust. Also, it is to be noted that the specific fuel consumption can be written as a function of the bypass ratio and specific thrust so that, for the ideal case, the specific fuel consumption becomes a function of the specific thrust only (for prescribed component efficiences, etc.). As a result of this simplified relationship, no obvious "optimum" choice of fan pressure ratio appears, as was the case for the separate-stream case. This writer, at least, was somewhat surprised by the (complete) independence of the required fan pressure ratio as a function of specific thrust for the ideal case, but it is apparent upon review that the presence of the optimal mixer compensates for changes in the fan pressure ratio and related changes in the turbine expansion ratio.

When losses are considered, similar manipulations show that the required bypass ratio is relatively insensitive to the value of fan pressure ratio. It is suggested, then, that the fan pressure ratio be selected as that necessary to provide a desired (input) value of the Mach number at the core stream entrance to the mixer (M_5) . In such a case, the performance variables as well as the required values for the fan bypass and fan pressure ratios, follow at once. It is to be noted that the mixer entrance conditions are chosen to be those that would lead to optimal performance of an ideal (no sidewall friction) constant-pressure mixer. This choice not only insures reasonably efficient mixer performance, but, through the static pressure mat-

0 2 1111115 7 8 9

Fig. 1 Mixed-stream turbofan station numbering.

ching requirement, establishes the fan stream mixer entry Mach number $(M_{3'})$ and allows simple estimation of the mixer outlet conditions.

Preliminary Manipulations

It is appropriate for preliminary design purposes to invoke several simplifying assumptions. Thus, the ratio of specific heats will be assumed locally constant, with values selected to be appropriate for the given component $(\gamma_c, \gamma_t, \gamma_g)$. Turbine cooling will not be included here although the approximate effect of turbine cooling is not difficult to estimate (Ref. 1, p. 243). Finally, the polytropic efficiencies of fan e_c , compressor e_c , and turbine e_t will be assumed constant.

Various groups defined in the Nomenclature are introduced for algebraic and computational convenience, thus B_i , C_i , and Γ_i . Note that the C_i and Γ_i reduce to unity when the ideal case is considered.

Conditions in the Mixer

Ideal (no sidewall friction) constant-pressure mixing will be assumed. If deemed appropriate, an additional loss can be included in the form of π_m , where typically a relationship could be expected of the form $\pi_m = 1 - KM_7^2$ in which K would be estimated empirically, but should be of the order of ≈ 0.05 .

In order to use the simple ideal mixing equations, $\gamma_7 = \gamma_9$ will be calculated as that γ necessary to create two streams of the same γ with the same combined stagnation enthalpy flux as the two entering streams at T_{15} and T_{13} . Thus, noting that $C_p = [\gamma/(\gamma-1)]R$ and assuming that the gas constants of all streams are equal, the stagnation enthalpy balance gives

$$\dot{m}_{c}(I+f)\frac{\gamma_{t}}{\gamma_{t}-1}T_{t5} + \dot{m}_{F}\frac{\gamma_{c}}{\gamma_{c}-1}T_{t3},$$

$$= \frac{\gamma_{g}}{\gamma_{g}-1}[\dot{m}_{c}(I+f)T_{t5} + \dot{m}_{F}T_{t3}]$$
(1)

from which it follows that

$$\gamma_9 = \gamma_c \frac{(1+f)\tau_{\lambda}\tau_t + \alpha\tau_r\tau_{c'}}{(1+f)(\gamma_c/\gamma_t)\tau_{\lambda}\tau_t + \alpha\tau_r\tau_{c'}}$$
(2)

The "optimal" results for the ideal constant-pressure mixer² give the mixer outlet conditions in terms of the inlet stagnation conditions (including the result that the optimal mixer is of constant area!). Prescription of the engine stream Mach number then allows determination of the required fan pressure ratio. The optimal results (Ref. 1, p. 160) can be manipulated to give the hierarchy of equations

$$M_{3'}^2 = M_5^2 \frac{S_{3'}}{\tau} \tag{3}$$

$$M_7^2 = M_3^2 \frac{(1+\alpha)S_{3'}}{S_{3'} + \alpha\tau} \tag{4}$$

$$\left(\frac{p_{t7}}{p_{t5}}\right)^{(\gamma_9-1)/\gamma_9} = S_{3'} \frac{1+\alpha\tau}{S_{3'}+\alpha\tau}$$
 (5)

also

$$\frac{\gamma_9 - 1}{2} M_5^2 = \frac{1 - 1/S_3}{1/\tau - 1} \tag{6}$$

Then, noting that

$$\tau = \frac{T_{t3'}}{T_{t5}} = \Gamma_I \frac{\tau_r \tau_{c'}}{\tau_\lambda \tau_t} \tag{7}$$

and, with some manipulation, that

$$1/S_{3'} = \left(\frac{p_{t3'}}{p_{t5}}\right)^{-(\gamma_g - I)/\gamma_g} = \frac{\tau_t}{\tau_{c'}} \tau_c C_I$$
 (8)

it follows from Eqs. (6-8) that

$$\tau_{c'}/\tau_t = 1/B_3 \tag{9}$$

The power balance between the fan, compressor, and turbine gives

$$\tau_{t} = 1 - \frac{1}{\eta_{m}(1+f)} \frac{\tau_{r}}{\tau_{\lambda}} [\tau_{c} - 1 + \alpha(\tau_{c'} - 1)]$$
 (10)

and combination of Eqs. (9) and (10) leads to an expression for the fan stagnation temperature ratio (and hence stagnation pressure ratio); thus

$$\tau_{c'} = \frac{1 - \left[1/\eta_m (1+f)\right] (\tau_r/\tau_\lambda) (\tau_c - 1 - \alpha)}{B_3 + \left[\alpha/\eta_m (1+f)\right] (\tau_r/\tau_\lambda)} \tag{11}$$

Nozzle Pressure Ratio

The nozzle pressure ratio may be written in terms of the stagnation pressure ratios of all the components as

$$\frac{p_{t9}}{p_0} = \pi_d \pi_r \pi_c \pi_b \pi_t \frac{p_{t7}}{p_{t5}} \pi_m \pi_n \tag{12}$$

Combination of Eqs. (5), (8), and (12) then leads to

$$\left(\frac{p_{t9}}{p_0}\right)^{(\gamma_9-1)/\gamma_9} = C_2 \frac{\tau_\lambda \tau_t + \alpha \Gamma_1 \tau_r \tau_{c'}}{\alpha \Gamma_1 C_1 + (\tau_\lambda / \tau_r \tau_c)}$$
(13)

It is to be noted that the expression for the ideal engine nozzle pressure ratio follows directly from Eq. (13) with $\Gamma_I = C_I = C_2 = 1$ and $\tau_I = \tau_{II}$. The expressions for the ideal values of various parameters are of use in the subsequent analysis and are listed here for convenience,

$$\left(\frac{p_{t9}}{p_0}\right)_I^{(\gamma_9-I)/\gamma_9} = \frac{\tau_\lambda \tau_{tI} + \alpha \tau_r \tau_{c'}}{\alpha + (\tau_\lambda/\tau_r \tau_c)}$$
(14)

Further, noting from Eq. (10) that

$$\tau_{II} = I - \frac{\tau_r}{\tau_{\lambda}} [\tau_c - I + \alpha(\tau_{c'} - I)]$$
 (15)

there follows

$$\frac{1}{1+\alpha} \frac{1}{\tau_r - I} \left(\tau_{\lambda} \tau_{tI} + \alpha \tau_r \tau_{c'} \right) \left[I - \left(\frac{p_0}{p_{tg}} \right)_I^{(\gamma_g - I)/\gamma_g} \right]$$

$$= I + \frac{1}{I+\alpha} \left(\frac{\tau_{\lambda}}{\tau_r \tau_c} - I \right) \left(\frac{\tau_r \tau_c - I}{\tau_r - I} \right) \tag{16}$$

Expression for the Bypass Ratio

In the ideal engine case, because the specific thrust is independent of the bypass pressure ratio, the expression for the thrust can be inverted to give the required bypass ratio. In what follows, the full equations are manipulated to provide the same result; but, in the case with losses, the equation for the bypass ratio α is itself weakly dependent on α . However, the resulting expression is easily solved by functional iteration. Routine cycle analysis leads to the expression (see Ref. 1, p. 152 for the development of the ideal form of this expression),

$$G = \left\{ \left(1 + \frac{f}{1+\alpha} \right) \left[\frac{(1+f)\tau_{\lambda}\tau_{t} + \alpha\tau_{r}\tau_{c'}}{(1+\alpha)(\tau_{r}-1)} \right] \times \left[1 - \left(\frac{p_{0}}{p_{t9}} \right)^{(\gamma_{g}-1)/\gamma_{g}} \right] \right\}^{\frac{1}{2}} - 1$$
(17)

so that with the definition of C_3 and Eqs. (16) and (17) there follows

$$G = \sqrt{C_3} \left[1 + \frac{1}{1+\alpha} \left(\frac{\tau_{\lambda}}{\tau_r \tau_c} - 1 \right) \left(\frac{\tau_r \tau_c - 1}{\tau_r - 1} \right) \right]^{\frac{1}{2}} - 1$$
 (18)

This expression is then inverted to obtain

$$\alpha = \frac{B_1}{[(G+I)^2/C_2] - I} - I \tag{19}$$

As stated above, the right side of this equation is weakly dependent on α (in the term C_3), but is easily solved by functional iteration.

The foregoing results may be grouped in a manner that allows straightforward solution. The equations, in SI units, are summarized in the following.

Summary

Inputs:

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$$h(J \cdot kg^{-1}), T_0(K), \gamma_t, \pi_d, \pi_b, \pi_n, \eta_b, \eta_m, e_c, e_{c'}, e_t,$$

$$K, \pi_c, \tau_\lambda, M_5, M_0, F/(\dot{m}_c + \dot{m}_F)(N \cdot s \cdot kg^{-1})$$

 $\gamma_c = 1.4$ is included within the equations

Outputs:

$$S(\text{mg}\cdot s^{-1}\cdot N^{-1}), \alpha, \pi_c$$

 $a_0 = 20.0\sqrt{T_0}$

Equations:

$$\tau_r = I + (M_0^2/5)$$

$$\tau_c = I/\pi_c^{3.5e_c}$$

$$G = \left(\frac{F}{\dot{m}_c + \dot{m}_F}\right) \frac{I}{a_0 M_0}$$

$$\Gamma_I = \frac{I}{3.5} \frac{\gamma_t}{\gamma_t - I}$$

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{(h\eta_b/1005T_o) - \tau_\lambda}$$

$$B_I = \left(\frac{\tau_\lambda}{\tau_r \tau_c} - I\right) \frac{\tau_r \tau_c - I}{\tau_r - I}$$

Table 1 Parameters of example

| Table 1 Farameters of example | | | | |
|--|-----------------|----------------------------------|--|--|
| $h = 4.42 \times 10^7 \text{ J} \cdot kg^{-1}$ | $\eta_b = 0.97$ | K=0 | | |
| $T_0 = 233 \text{ K}$ | $\eta_m = 0.99$ | $\pi_c = 15$ | | |
| $\gamma_t = 1.3$ | $e_c = 0.90$ | $\tau_{\lambda} = 7$ | | |
| $\pi_b = 0.95$ | $e_{c'} = 0.90$ | $\pi_d = 0.87, 0.90, 0.93, 0.96$ | | |
| $\pi_n = 0.99$ | $e_t = 0.90$ | $M_5 = 0.1, 0.4, 1.0$ | | |

Begin iteration with
$$C_3 = 1$$
, $\tau_{c'} = 1.2$, and $\pi_m = 1$,
$$\alpha = \frac{B_I}{[(G+I)^2/C_3] - I} - I$$

$$\tau_t = I - \frac{I}{\eta_m (I+f)} \frac{\tau_r}{\tau_\lambda} [\tau_c - I + \alpha (\tau_{c'} - I)]$$

$$B_2 = (I+f)\tau_\lambda \tau_t + \alpha \tau_r \tau_{c'}$$

$$\gamma_9 = \frac{1.4B_2}{(I+f)(I.4/\gamma_t)\tau_\lambda \tau_t + \alpha \tau_r \tau_{c'}}$$

$$\Gamma_2 = 3.5(\gamma_9 - I/\gamma_9)$$

$$\Gamma_3 = \Gamma_I \Gamma_2$$

$$C_1 = \pi_b^{(\gamma_9 - I)/\gamma_9} \tau_c^{(\Gamma_2 e_c - I)} \tau_c^{(\Gamma_3/e_t - I)} \tau_c^{(I-e_c \cdot \Gamma_2)}$$

$$C_2 = (\pi_d \pi_b \pi_m \pi_n)^{(\gamma_9 - I)/\gamma_9} \tau_r^{(\Gamma_2 - I)} \tau_c^{(\gamma_c \Gamma_2 - I)}$$

$$\times \tau_t^{(\Gamma_3/e_t - I)}$$

$$B_3 = \frac{I + [(\gamma_9 - I)/2]M_3^2}{[(\gamma_9 - I)/2]M_3^2(I/\Gamma_I)(\tau_\lambda/\tau_r) + \tau_c C_I}$$

$$\tau_{c'} = \frac{I - [I/\eta_m (I+f)](\tau_r/\tau_\lambda)(\tau_c - I - \alpha)}{B_3 + [\alpha/\eta_m (I+f)](\tau_r/\tau_\lambda)}$$

$$\left(\frac{p_0}{p_{t9}}\right)^{(\gamma_9 - I)/\gamma_9} = \frac{\alpha \Gamma_I C_I + (\tau_\lambda/\tau_r \tau_c)}{C_2 \{\tau_\lambda \tau_t + \alpha \Gamma_I \tau_r \tau_{c'}\}}$$

$$C_3 = \frac{[I + (f/I + \alpha)]B_2[I - (p_0/p_{t9})^{(\gamma_9 - I)/\gamma_9}]}{(\tau_r - I)(I + \alpha + B_I)}$$

All quantities are now updated and iteration continued until the desired accuracy is obtained. Then

 $M_7^2 = M_5^2 \left[\frac{1+\alpha}{1+\alpha\Gamma_1 C_1 (\tau_1 \tau_2/\tau_1)} \right]$

 α = as already calculated

 $\pi_m = I - KM_7^2$

$$\pi_{c'} = \tau_c^{3.5e_{c'}}$$

$$S = \frac{f(10^6)}{(1+\alpha) [F/(\dot{m}_c + \dot{m}_F)]}$$

Example Results

The method is appropriate for any mixed-flow turbofan, but as an example an engine suitable for use in a high-performance "supercruiser" aircraft is considered. The engine is to have a specific thrust, $F/(\dot{m}_c + \dot{m}_F)$, of 300 N·s·kg⁻¹ (without afterburning) at a flight Mach number M_θ of 2. The other parameters chosen are listed in Table 1.

Table 2 $F/(\dot{m}_c + \dot{m}_F) = 300 \text{ N} \cdot \text{s} \cdot \text{kg}^{-1} (30.6 \text{ lbf} \cdot \text{s/lbm})$

| Turbofan | α | 7 | S, $mg \cdot s^{-1} \cdot N^{-1}$ (1bm/h·1bf) | |
|-------------------|-------|------------|--|---------|
| | u | $\pi_{c'}$ | (10111) | 11-101) |
| Optimal separate- | | | | |
| stream | 0.293 | 2.85 | 40.2 | (1.42) |
| Mixed-stream | • | | | |
| $M_5 = 0.1$ | 0.310 | 1.69 | 39.7 | (1.40) |
| $M_5 = 0.4$ | 0.310 | 1.78 | 39.7 | (1.40) |
| $M_5 = 1.0$ | 0.308 | 2.25 | 39.7 | (1.40) |

Table 2 and Fig. 2 show the behavior of the specific fuel consumption, bypass ratio, and bypass pressure ratio for a separate-stream and for a mixed-stream turbofan. In these figures, $\pi_d = 0.93$ and the separate-stream case corresponds to the "optimal" configuration (α and $\pi_{c'}$ giving minimum S).

The effect of variations in inlet efficiency π_d is shown in Figs. 3 and 4. In these figures, M_5 for the mixed-stream turbofan is taken to be 0.4.

Discussion

It is evident from Table 2 that, as claimed in the introduction, the performance of the mixed-stream turbofan and the

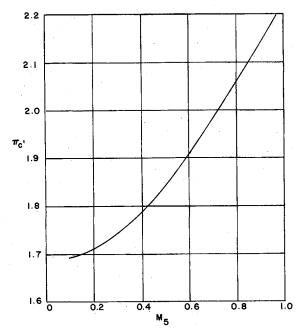


Fig. 2 Variation of bypass pressure ratio with mixer inlet Mach

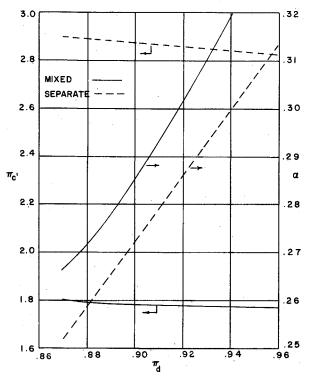


Fig. 3 Variation of bypass ratio and bypass pressure ratio with inlet efficiency.

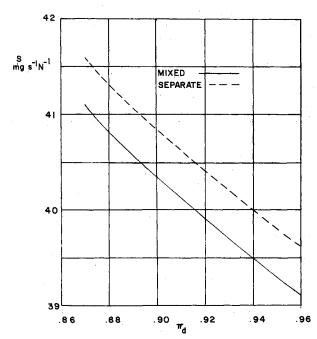


Fig. 4 Variation of specific fuel consumption with inlet efficiency.

required bypass ratio are relatively insensitive to the fan pressure ratio. Thus, these calculations for the mixed-stream case, with realistic values for component losses, show that for this case with fixed specific thrust, the specific fuel consumption remains constant to three significant figures and the design bypass ratio varies less than 1% for a range of design fan pressure ratios of 1.69-2.25. The small difference between the specific fuel consumption of the mixed-stream turbofan and the "optimal" separate-stream turbofan has little significance for such approximate calculations (note, for example, that the additional mixer loss parameter K was taken to be zero).

A result of greater significance, however, is evident in the much lower bypass pressure ratios identified with the lower values of M_5 , that exist for the mixed-flow turbofan to provide essentially the same performance as the separate-stream turbofan. Two aspects of this result are important. It is evident that if an engine were designed to perform optimally with separate exhaust streams and if it were then to be adapted for mixed-stream use, the mixer design would be problematical. That is, either a very high Mach number mixer would be required or the mixer would have to operate with entrance conditions far from optimum, with consequent substantial penal-

ty in stagnation pressure. Perhaps the more important aspect of the indicated results, however, is that if a designer knew before hand that his engine was always to be used in a mixed-stream installation, he could exploit the presence of the mixer to employ a fan of lower pressure ratio with consequent requirement of fewer stages, lower rotational speed, or less advanced technology in both the fan and turbine.

The behavior of α , $\pi_{c'}$, and S with variation in π_d reflect the expected trends. Thus, it is to be expected that if the inlet efficiency increases, the optimal bypass ratio would also increase, a reflection of the fact that the related fan pressure ratio necessary to provide the required specific thrust decreases and hence more enthalpy extraction from the turbine is available to pressurize a larger fan stream. Obviously, it would be expected also that a component efficiency increase would lead to a decrease in specific fuel consumption, and such is the case.

Summary and Conclusions

By utilizing optimal solutions and appropriate simplifications, a simple equation set is derived to describe the design performance of mixed-stream turbofans when component losses are included. The resulting equation set is easily programmed on a personal computer or programmable calculator.

Example results illustrate several expected design trends. A result of note, however, is that the configuration of an optimal separate-stream turbofan can be markedly different (in bypass pressure ratio) from a mixed-stream turbofan having an optimal mixer. A related, somewhat surprising result is that for a given specific thrust and a substantial range of fan pressure ratios, the specific fuel consumption of the mixed-stream configuration is very nearly independent of the fan pressure ratio, provided an optimal mixer is used.

As a final point, it is to be noted that if a turbofan is to be designed to take advantage of the presence of a mixer in the manner suggested herein, it is of great importance that the mixer be of sound design. In a conventional installation, there is little penalty for incomplete mixing, because the conventional installation is near optimal for the unmixed case. This would not be the case if, as suggested here, a relatively low fan pressure ratio was employed. As a result, it is imperative that near complete mixing be attained.

References

¹Oates, G. C., "Aerothermodynamics of Gas Turbine and Rocket Propulsion," *AIAA Education Series*, J. Przemieniecki (series ed.) AIAA, New York, 1984.

²Knox, R. M., "A Study of Optimized Constant Pressure Jet Pumps," Marquart Corp. Rept. 5827A, Dec. 30, 1960, rev. March 30, 1962.